Maxima and Minima

In this section we will study problems where we wish to find the maximum or minimum values of a functions and also show how differentiation can be used to find the maximum and minimum values of a function. Because the derivative provides information about the gradient or slope of the graph of a function we can use it to locate points on a graph where the gradient is zero. We shall see that such points are often associated with the largest or smallest values of the function, at least in their immediate locality.

Finding a maximum or a minimum of a function is clearly an important in everyday experience such as

* A manufacturer wants to maximize her profits.
* A businessman wish to minimize the cost of production.
* A contractor wants to minimize his costs subject to doing a good job.
* A physicist wants to find the wavelength that produces the maximum intensity of radiation.

There are two types of maxima and minima of a functions:

* Absolute ( Global ) maxima and Absolute (Global) minima
* Local (Relative) maxima and Local (Relative) minima.

Note:

1. Maxima is the plural form of maximum
2. Minima is also the plural form of maximum
3. Extrema is the plural form of extremun
4. Maxima + Minima=Extrema and Maximum + Minimum=Extremum

**Some Special points on a curve:**

**Critical point:** A point on a curve in which derivative is zero or function is not differentiable.

**Saddle point:** A saddle point is a point on a curve where second order derivative zero or undefined.

**Stationary point:** A stationary point is a point on the curve where gradient of a function is zero. If gradient of the curve changes sign at stationary point then it called turning point otherwise horizontal Inflection.

**Definition:** A function is said to have a maximum for a value  of ifis greater than any other value that the function can have in the small neighborhood of.

Similarly, a function is said to have a minimum for a value  of ifis less than any other value that the function can have in the small neighborhood of.

maximum

minimum

**Theorem (Fermat's Theorem):**If has a local maximum or minimum at , and if exists,

then.

**Working rule for finding maxima and minima:**

If function be given, find and equate it to zero. Solve this equation for.

Let its roots be, … … ….

Find and hence find …. …. …. …..

* If is negative we have a maximum at .
* If is positive we have a minimum at .
* If find and then .
* If , then there is neither maxima nor minima at 

If ; find and then .

If is negative, then is maximum and if is positive, then is minimum at.

If Then find and so on.

* 
* if be odd, then there is neither maxima nor minima at 
* if be even and if is negative then is maximum at and
* ifis positive then is minimum at .

**Mathematical problems**

**Problem 01:** Find the maximum and minimum values of .

**Solution:**

Let 

Differentiating with respect to *x* we get,



We know that for maximum and minimum values,













Again, differentiating eq.(1) with respect to *x* we get,



For  we get,



Therefore, the given function is maximum at 

The maximum value is,



For  we get,



Therefore, the given function is minimum at 

The minimum value is,



For  we get,



Therefore the test fails.





Therefore, the given function is neither maximum nor minimum at ****

**Problem 02: Find the extremmum values of .**

**Solution:**

****

Differentiating with respect to *x* we get,

****



We know that for maximum and minimum values,













Again, differentiating eq.(1) with respect to *x* we get,

****

For  we get,



Therefore, the given function is maximum at 

The maximum value is,



For  we get,

****

Therefore, the given function is minimum at 

The minimum value is,

 (*Ans*).

**Problem 03:** Find the maximum and minimum values of 

**Solution:**

****

Differentiating with respect to *x* we get,

****

We know that for maximum and minimum values,













Again, differentiating eq.(1) with respect to *x* we get,

****

For  we get,



Therefore, the given function is minimum at 

The minimum value is,



For  we get,

****

Therefore, the given function is maximum at 

The maximum value is,



**Try Yourself:**

1. Find the maximum and minimum values for .
2. Find the maximum and minimum values for .
3. Find the maximum and minimum values for .
4. Find maximum or minimum value of the function.
5. Find the saddle point , critical and point of inflection for 
6. Find maxima and minima of the function of the function 
7. Find maxima and minima of the function of the function 